Let $f(x) = \dfrac{1}{x \ln x}$.

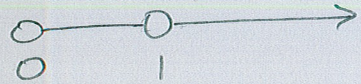
SCORE: _____ / 45 PTS

[a] Find all intervals on which f is continuous.

X IS CONT. ON $\mathbb{R}$

$\ln x$ IS CONT. ON $(0, \infty)$

$x \ln x \neq 0$ IF $\ln x \neq 0$ IE. $x \neq 1$ 5



$(0,1)$, $(1,\infty)$

5 5

[b] Find the limit of f at each discontinuity.
Each limit should be a number, ∞ or $-\infty$. **<u>Write DNE only if the other possibilities do not apply.</u>**

$\lim\limits_{x \to 1^-} \dfrac{1}{x \ln x} = -\infty$ 6 $\qquad \dfrac{1}{1 \cdot 0^-}$

$\lim\limits_{x \to 1^+} \dfrac{1}{x \ln x} = \infty$ 6 $\qquad \dfrac{1}{1 \cdot 0^+}$

SO $\lim\limits_{x \to 1} \dfrac{1}{x \ln x}$ DNE 3

($x=0$ IS NOT A DISCONTINUITY SINCE f IS NOT DEFINED IN AN OPEN INTERVAL AROUND $x=0$)

[c] State the type of each discontinuity in [b].

3 $x=0$ INFINITE DISCONTINUITY

[d] Find the equations of all horizontal asymptotes of f.

3 $\lim\limits_{x \to -\infty} \dfrac{1}{x \ln x}$ DNE SINCE f IS NOT DEFINED FOR $x < 0$

6 $\lim\limits_{x \to \infty} \dfrac{1}{x \ln x} = 0$

$\dfrac{1}{\infty \cdot \infty} \to \dfrac{1}{\infty}$

HORIZONTAL ASYMPTOTE

$y=0$ 3

State the formal definition of "jump discontinuity". **SCORE: _____ / 10 PTS**

f HAS A JUMP DISCONTINUITY AT a IF 2

$\lim_{x \to a^-} f(x)$ AND $\lim_{x \to a^+} f(x)$ EXIST BUT $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$

4 4

State the Squeeze Theorem. 3

SCORE: _____ / 10 PTS

IF $f(x) \le g(x) \le h(x)$, FOR ALL X IN AN OPEN INTERVAL AROUND a, EXCEPT POSSIBLY AT a,

2

AND $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$

3

THEN $\lim_{x \to a} g(x) = L$ 2

At time t hours, the position of an object moving along a line is $s(t) = 4t^2 - t^3$ kilometers. **SCORE: _____ / 20 PTS**

Find the instantaneous velocity of the object at time $t = 2$. **<u>Give the units of your answer.</u>**

$$\lim_{b \to 2} \frac{s(b) - s(2)}{b - 2}$$

$$= \boxed{\lim_{b \to 2} \frac{4b^2 - b^3 - 8}{b - 2}} \quad 7$$

$$= \boxed{\lim_{b \to 2} (-b^2 + 2b + 4)} \quad 7$$

$$= -4 + 4 + 4 = \boxed{4} \; \boxed{\frac{km}{hr}} \quad 2$$

4

$$\begin{array}{r|rrrr} 2 & -1 & 4 & 0 & -8 \\ & & -2 & 4 & 8 \\ \hline & -1 & 2 & 4 & \boxed{0} \end{array}$$

If $f(x)=\frac{x}{3-2x}$, find $f'(x)$. **SCORE: _____ / 20 PTS**

$$\boxed{\lim_{h\to 0}\frac{\frac{x+h}{3-2(x+h)}-\frac{x}{3-2x}}{h}}\cdot\frac{(3-2(x+h))(3-2x)}{(3-2(x+h))(3-2x)}$$

$$=\boxed{\lim_{h\to 0}\frac{(x+h)(3-2x)-x(3-2x-2h)}{h(3-2x-2h)(3-2x)}}$$

$$=\boxed{\lim_{h\to 0}\frac{\cancel{3x}+3\cancel{h}-\cancel{2x^2}-\cancel{2xh}-\cancel{3x}+\cancel{2x^2}+\cancel{2xh}}{\cancel{h}(3-2x-2h)(3-2x)}}$$

$$=\boxed{\lim_{h\to 0}\frac{3}{(3-2x-2h)(3-2x)}}=\boxed{\frac{3}{(3-2x)^2}}$$

4 EACH

Find a function f and a **non-zero** number a such that the derivative of f at a is given by SCORE: _____ / 15 PTS

$$\lim_{h\to 0}\frac{4\arctan(h-1)+\pi}{h} = \lim_{h\to 0}\frac{4\arctan(-1+h)-(-\pi)}{h} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

Show that your answers are correct using the definition of the derivative at a point.

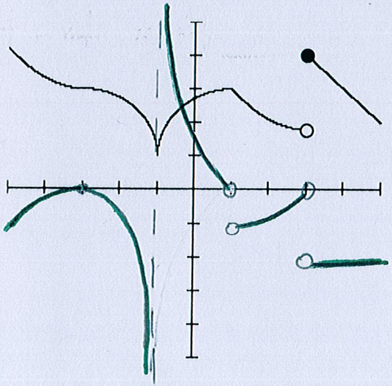
4 $f(x) = 4\arctan x$, $a = -1$ 4

$$\lim_{h\to 0}\frac{4\arctan(-1+h)-4\arctan(-1)}{h}$$ 4

$$= \lim_{h\to 0}\frac{4\arctan(h-1)-4(-\frac{\pi}{4})}{h}$$

$$= \lim_{h\to 0}\frac{4\arctan(h-1)+\pi}{h}$$ 3

The graph of $f(x)$ is shown below. Sketch a graph of $f'(x)$ on the same axes. **SCORE: _____ / 15 PTS**



Prove that the equation $x^2 = 1 + \sqrt[3]{x}$ has a solution in the interval $(-1, 1)$. **SCORE: _____ / 15 PTS**

$x^2 - \sqrt[3]{x} = 1$

LET $f(x) = x^2 - \sqrt[3]{x}$

$1\frac{1}{2}$

f IS CONTINUOUS ON $\mathbb{R}$ SINCE x^2, $\sqrt[3]{x}$ ARE BOTH CONTINUOUS ON $\mathbb{R}$

AND SO IS THEIR DIFFERENCE $1\frac{1}{2}$

$\boxed{\begin{array}{l} f(-1) = 1 - (-1) = 2 \\ f(1) = 1 - 1 = 0 \end{array}}$

2 EACH EXCEPT AS NOTED

$f(1) < 1 < f(-1)$

BY IVT, $f(x) = x^2 - \sqrt[3]{x} = 1$ FOR SOME $x \in (-1, 1)$

IE. $x^2 = 1 + \sqrt[3]{x}$